Algorithms, Incentives, and Multidimensional Preferences

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Algorithms and Incentives

**Past:** Algorithms as black box

![Diagram](image)

**Now:** Algorithm as Platform

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Examples:
- Routing Protocols
- Crowdsourcing
- Electronic Commerce, Sharing Economy

Design requirement: Consider user incentives
Algorithms and Incentives

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Input → Algorithm → Output

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Revenue Maximizing Mechanisms

**ISP service:**
- High quality vs. low quality

Chen et al., 2015: computationally hard

Theorem (Haghpanah, Hartline, 2015)
- If types with high $v_H$ are less sensitive
  - Only offering high quality optimal

$v_L = \frac{3}{4}$
Revenue Maximizing Mechanisms

ISP service:
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How should the services, and lotteries over them, be priced?
Revenue Maximizing Mechanisms

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- Distribution $f: (v_H, v_L) \sim f$
- **Goal:** maximize expected revenue

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*If types with high \( v_H \) are less sensitive \( \Rightarrow \) Only offering high quality optimal*
Technique

**Reduce** the **average-case** problem to a **point-wise** problem

**Lemma (Haghpanah, Hartline, 2015)**

There exists a virtual value function $\phi$ such that

1. Revenue of any mechanism $= \mathbb{E}_v [x(v) \cdot \phi(v)]$

2. Selling only high quality maximizes $x(v) \cdot \phi(v)$ pointwise.

**Idea:** for any covering of space $\gamma$, there exists $\phi_{\gamma}$ such that

$\text{Revenue of any mechanism } = \mathbb{E}_v [x(v) \cdot \phi_{\gamma}(v)]$

**Challenge:** find $\gamma$ such that $\phi_{\gamma}$ satisfies second property.

(1, 0) (0, 0) allocation covering $\gamma$ (paths) virtual value $\phi_{\gamma}$ virtual value $\phi_{\gamma}$
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\[ \begin{array}{c}
\text{allocation} \\
(0, 0) \quad (1, 0) \\
\hline
\text{virtual value } \phi^\gamma \\
\hline
\text{covering } \gamma \text{ (paths)}
\end{array} \]