Mathematical Methods in Computer Science*
Lecture 0: How to write the lecture notes

Notes taken by Boaz Barak

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Summary: We explain how to write the lecture notes for the course "Mathematical Methods in Computer Science" taught by Gil Kalai and Avi Wigderson. The lecture notes are part of the course requirement and will comprise 50% of the final grade. **You should submit the notes to me by email (`boaz@wisdom.weizmann.ac.il`) not later than 2 weeks after the lecture.**

1 What should the notes contain?

The notes should contain at least all the material presented in the lecture. You don't have to follow the exact way in which the material was presented. Important points:

1. The notes should contain full proofs even if in class the lecturer skipped some parts or only provided proof sketches.
2. The lecture notes should contain not only theorems and proofs but also high level comments and explanations.
3. The lecture notes should also contain all the exercises given in the lecture.
4. If you have any problems with understanding part of the material in class **don't hesitate to ask either the lecturer (Gil or Avi) or me for explanations and clarifications**.

For an example of the expected level of lecture notes, see the lecture notes for the course "Expander Graphs and their Applications" of Nati Linial and Avi Wigderson on `http://www.wisdom.weizmann.ac.il/~boaz/ExpanderCourse/index.html`, or the lecture notes for the Complexity course of Oded Goldreich on `http://www.wisdom.weizmann.ac.il/~oded/cc99.html`.

* Lecture Notes for a course given by Gil Kalay and Avi Wigderson at the Hebrew University, Israel.

2 Basic steps in writing the notes

I assume you know how to use LaTeX. If not, see the next section.

1. Download the files `lecture0.tex` , `lnotes.sty` and `lecture0.bib` from the course's homepage (`http://www.wisdom.weizmann.ac.il/~boaz/methods2003/index.html`) and put them in the same directory.

2. Rename the files `lecture0.tex` and `lecture0.bib` to `lecture`x`.tex` and `lecture`x`.bib`, where x is your lecture number.

3. Open the file `lecture`x`.tex` and do the following:

   - Change the definitions at the top of the files to your name(s), the lecture number, and the title of the lecture.
   - Delete all the lines between the lines marked by

     ```
     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
     % BEGIN BODY of Document
     ```

     and the lines marked by

     ```
     % END BODY of document
     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
     ```

   - Write your own text in this part. Start with a summary of the lecture – put it inside `\begin{summary}` . . . `\end{summary}`.

3 If you are new to LaTeX

In the **links** section of my homepage (`http://www.wisdom.weizmann.ac.il/~boaz`) there are links both for LaTeX software to download and tutorials on LaTeX. Especially recommended are the "Not so short introduction to LaTeX2ϵ" tutorial and the Indian TeX Users Group LaTeX tutorial.

4 Conventions and notations

Because the lecture notes are all going to be merged together into one document, you need to follow the following conventions:

Naming Your file should be named `lecture`x`.tex`, where x is the lecture number.

Labels When you use a label for a theorem, definition, etc.., prefix the label with your initials. For example, if I want to give a label to a theorem I will use a label such as `\label{BB:thm:PneqNP}`. When you refer to a theorem use the command `\theoremref{}` instead of `Theorem~\ref{}`. There are similarly defined commands such as `\defenitionref{}` , `\exerciseref{}` etc.. (e.g., Theorem 5.2 , Algorithm 5.3).

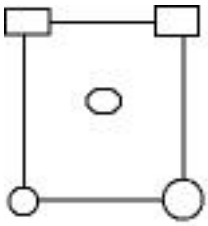

Figure 1: An example for including graphics

Graphics If you wish to include graphics in your presentation you should prepare the file in *both* `.eps` and `.jpg` format. You should name give the two files the same name but different extension, and the name should start with the prefix `lec`x`_`. The command to include graphics is `\includegraphics*{filename}`. For example, see Figure 1 where the graphics were included using the command `\includegraphics*{lec0_example}` (where the graphic files are `lec0_example.eps` and `lec0_example.jpg`. For more information about this command see the reference manual for the `graphicx` LaTeX package which can be found on `http://www.cmis.csiro.au/Graham.Williams/TeX/docs/grfguide.pdf`.

Private macros you should prefix any new LaTeX command you define with your initials. For example, if I wanted to define a macro for the transpose operator, it would be `\newcommand{BBtrans}[1]{#1^T}`. Before defining a new command, see if an equivalent command is not already defined below or in the AMS packages.

Notations Use the following notations:

- Use $x \leftarrow_{\mathrm{R}} S$ (`$x \getsr S$`) to denote that x is chosen at random from S where S is either a set or a distribution.
- Use U_n to denote the uniform distribution on strings of length n (i.e., on $\{0,1\}^n$).

4.1 References and bibligoraphy

You should use BibTeX for references. Whenever you want to cite a paper you should use the command `\cite{key}` where `key` consists of last name of the first author, the first two letters of all other authors, and the last two digits of the publication year. For example, to cite a 1981 paper by Frankl and Wilson you should use `\cite{FranklWi81}`. The result is [FW81]

You should then find a BibTeX entry for the paper and place it in your `lecture`x`.bib` file (where x is the lecture name). Don't forget to change the key to the formal prescribed above. For example, the BibTeX entry for the Frankl and Wilson paper [FW81] is:

```
@article {FranklWi81,
    AUTHOR = {Frankl, P. and Wilson, R. M.},
     TITLE = {Intersection theorems with geometric consequences},
   JOURNAL = {Combinatorica},
  FJOURNAL = {Combinatorica. An International Journal of the J\'anos Bolyai
              Mathematical Society},
    VOLUME = {1},
      YEAR = {1981},
    NUMBER = {4},
     PAGES = {357--368},
      ISSN = {0209-9683},
     CODEN = {COMBDI},
   MRCLASS = {05C35 (05A17 05A20 05C15)},
  MRNUMBER = {84g:05085},
MRREVIEWER = {E. C. Milner}, }
```

Once you do this, and run LaTeX on the `.tex` file, BibTeX on the `.bib` file, and then again LaTeX twice on the `.tex` file, the bibliography will be added automatically.

Finding BibTeX entries. you can find BibTeX entries for papers on the web. Two good places are:

- The computer science bibliography: `http://liinwww.ira.uka.de/bibliography/`
- AMS MR lookup: `http://www.ams.org/mrlookup`

5 Useful macros that are predefined for you

The template[1] already contains the following LaTeX commands and environments. If you want to define additional commands, you need to prefix them with the initials of your name.

[1] Actually, these are defined in the file `lnotes.sty` which you can view but SHOULD NOT MODIFY!.

5.1 Math Symbols (partial list)

`\eqdef` : $\stackrel{def}{=}$		
`\N` : $\mathbb{N}$	`\R` : $\mathbb{R}$	`\Z` : $\mathbb{Z}$
`\C` : $\mathbb{C}$	`\F` : $\mathbb{F}$	
`\getsr` : $\leftarrow_{\text{R}}$	`\st` : s.t.	`\Ex` : $\mathbb{E}$
`\e` : ϵ		
`\To` : $\to$		
`\ceil{x}` : $\lceil x \rceil$	`\floor{x}` : $\lfloor x \rfloor$	`\angles{x,y,z}` : $\langle x, y, z \rangle$
`\norm{x}{\infty}` : $\|x\|_\infty$	`\normone{x}` : $\|x\|_1$	`\normtwo{x}` : $\|x\|_2$
`\dprod{x}{y}` : $\langle x, y \rangle$	`\bits` : $\{0,1\}$	
`\poly` : poly	`\polylog` : polylog	
`\GF` : GF	`\charfun{S}` : $\mathbf{1}_S$	

In addition, all the AMSLaTeX macros are available. Particularly useful macros are `\binom{}{}` for the binomial coefficient (e.g. $\binom{n}{k}$), `\pmod{}` for modular equations (e.g., $2 = 9 \pmod 7$), `\tfrac{}{}` for fractions that take less vertical space (e.g. $\tfrac{3}{4}$), and `\vec{}` for vectors (e.g., $\vec{v}$). You can find more information about AMSLaTeX in the tutorials mentioned above and in the AMSLaTeXuser guide.

5.2 Environments

List of environments:

- Theorems etc.: **theorem** , **claim** , **subclaim** (for a claim inside a proof of a theorem) , **lemma** , **corollary** , **conjecture** , **observation**.
- Definitions etc.: **definition** , **construction**, **example** , **remark**
- Exercises etc.: **exercise** and **answer**

Some examples:

Definition 5.1. A function $f : \{0,1\}^n \to \{0,1\}$ is *balanced* if

$$\Pr_{x \leftarrow_{\text{R}} \{0,1\}^n}[f(x) = 1] = \frac{1}{2}$$

Theorem 5.2. *For every $\alpha \in \{0,1\}^n$, let $f_\alpha : \{0,1\}^n \to \{0,1\}$ denote the following function $f_\alpha(x) = \langle x, \alpha \rangle$. Then, f_α is balanced.*

Algorithm 5.3 (Computing a square root).
Input: $n \in \mathbb{N}$

1. Let $l \leftarrow 0$, $h \leftarrow n$.
2. Do the following while $h > l$:
   (a) Let $m \leftarrow \lfloor \frac{l+h}{2} \rfloor$.
   (b) If $m^2 < n$ then let $l \leftarrow m$. Otherwise, let $h \leftarrow m$.

3. Output m.

Which were produced by

```
\begin{definition} \label{BB:def:bal} A function $f:\bits^n \To
\bits$ is \emph{balanced} if
\[
\Pr_{x \getsr \bits^n}[  f(x)= 1 ] = \frac{1}{2}
\]
\end{definition}

\begin{theorem} \label{BB:thm:prod} For every $\alpha \in \bits^n$,
let $f_{\alpha}:\bits^n \To \bits$ denote the following function
$f_{\alpha}(x) = \dprod{x}{\alpha}$. Then, $f_{\alpha}$ is
balanced.
\end{theorem}

\begin{algorithm}[Computing a square root] \label{BB:alg:sqroot}
\textbf{Input:} $n \in \N$

\begin{enumerate}

\item Let $l \leftarrow 0$, $h \leftarrow n$.

\item Do the following while $h>l$:

\begin{enumerate}
\item  Let $m \leftarrow
\floor{\tfrac{l+h}{2}}$.

\item If $m^2 < n$ then let $l \leftarrow m$. Otherwise, let $h
\leftarrow m$.

\end{enumerate}

\item Output $m$.
\end{enumerate}

\end{algorithm}
```

References

[FW81] P. Frankl and R. M. Wilson. Intersection theorems with geometric consequences. *Combinatorica*, 1(4):357–368, 1981.